Homework #5

Math 231b

"Due": April 26th, 2017

Guidelines:

- Type up your solution to the assignment in LaTeX. (You might want to avail yourself of the excellent diagrams package `tikz-cd`.)
- Submit the PDF via Canvas, in the Assignments section.

Failure to meet these guidelines may result in loss of points.[1]

Problem 1. Suppose you believe in complex Bott periodicity, so that the homotopy groups of $BU(n)$ have the form $\pi_{\mathrm{odd}} BU(n) = 0$ and $\pi_{\mathrm{even}} BU(n) = \mathbb{Z}$ in the range $[0, 2n]$. Set $n = 3$ and describe the action of the Steenrod algebra on $H^*(BU(3); \mathbb{F}_2)$. Then try $n = 4$. Then $n = 5$. Stop once you get sick of the exercise.

Problem 2. Return to the picture of the Adams spectral sequence computing $\pi_* ko$ described in class. At a glance, it appears that there could be a potential differential $d_r h_1 = h_0^{r+1}$. *Without* assuming Bott periodicity, argue why this differential cannot occur. (Hint: $h_0 h_1 = 0$.)

Problem 3. Compute the first several terms (until you get tired) of a free resolution of $\mathbb{F}_2$ as a module over the Steenrod algebra. (To check your answer, you can find a considerable chunk of such a resolution on page 85 of this PDF: `https://www.math.cornell.edu/~hatcher/AT/ATch5.pdf`.) Once you have the resolution, use it to compute Ext and compare your answer with the part of the Adams spectral sequence drawn in class.

Problem 4. Let $E(1)$ denote the exterior $\mathbb{F}_2$–algebra on two generators e_1 and e_3, of degrees 1 and 3 respectively. Calculate $\mathrm{Ext}^{*,*}_{E(1)}(\mathbb{F}_2, \mathbb{F}_2)$.

Task 5. Try to read Section 8 of Steve Wilson's *Brown–Peterson Homology: An Introduction and Sampler*. He gives a calculation of the mod–p Steenrod algebra there—try to convert it into a calculation of the mod–2 Steenrod algebra, which simplifies his discussion considerably. (The space he calls $\underline{K}_q$ is what we are calling $K(\mathbb{F}_p, q)$.) (You shouldn't need any of the meat of the previous 7 sections to read this—except for the definition of a "Hopf ring", which is at the start of Section 7.)

Problem 6. Figure out both the statement and the proof of the 5–Lemma and the Snake Lemma in mod–$\mathcal{C}$ homological algebra.

[1] This version of the assignment was compiled on April 12, 2017.