Discrete Mathematics

Basic Structures:
Sets, Functions, Sequences, and Sums

Prof. Steven Evans

2.1: Sets

Sets

Definition

A *set* is an unordered collection of objects, called *elements* or *members* of the set. A set is said to *contain* its elements. We use "$a \in A$" to denote that a is an element of the set A. Likewise, we use "$a \notin A$" to denote that a is not an element of A.

Set-builder notation

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Example

$\mathbb{Q}^+ = \{x \in \mathbb{R} \mid x = \frac{p}{q} \text{ for positive integers } p \text{ and } q\}$

Sets

More examples of sets

- $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$, the set of *natural numbers*.
- $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, the set of *integers*.
- $\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$, the set of *positive integers*.
- $\mathbb{Q} = \{\frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{Z}, q \text{ nonzero}\}$, the set of *rational numbers*.
- $\mathbb{R}$, the set of *real numbers*.
- $\mathbb{R}^+$, the set of *positive real numbers*.
- $\mathbb{C}$, the set of *complex numbers*.

Intervals

Definition

When a and b are real numbers with $a < b$, we write:

- $[a, b] = \{x \mid a \leq x \leq b\}$,
- $[a, b) = \{x \mid a \leq x < b\}$,
- $(a, b] = \{x \mid a < x \leq b\}$,
- $(a, b) = \{x \mid a < x < b\}$.

Sets

Definition

Two sets are said to be *equal* when they have the same elements, meaning that A and B are equal if and only if the sentence

$$\forall x(x \in A \leftrightarrow x \in B)$$

holds. We write $A = B$ for this sentence.

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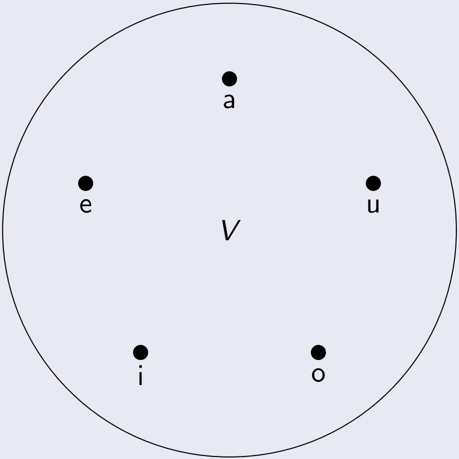
Definition

There is a special set that contains no elements. This set is called the *empty set* or the *null set* and is denoted by $\varnothing$ or by $\{\}$.

Venn Diagrams

Example

A Venn diagram for the set of vowels:



Subsets

Definition

The set A is a *subset* of B if every element of A is also an element of B. This is written symbolically as

$$\forall x(x \in A \to x \in B).$$

We use the notation $A \subseteq B$ to indicate this relationship.

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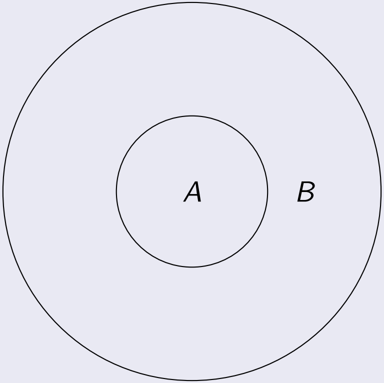
Note

Note that to show that A is *not* a subset of B, we need only find one element $x \in A$ with $x \notin B$. This element is a counterexample to the claim that $x \in A$ implies $x \in B$.

Subsets

"Subset" as a Venn diagram



Subsets

Proper subsets

When we want to emphasize that $A \subseteq B$ but $B \neq A$, we write $A \subset B$ and say that A is a *proper subset* of B. For $A \subset B$ to be true, it must be the case that $A \subseteq B$ yet there is an element of B which is not an element of A.

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Double containment

A useful way to show that two sets are equal is to show that each is a subset of the other. In other words, if we can show $A \subseteq B$ and $B \subseteq A$, then $A = B$.

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$$\begin{aligned}&(\forall x(x \in A \leftrightarrow x \in B)) \\ &\quad \leftrightarrow (\forall x(x \in A \to x \in B) \wedge \forall x(x \in B \to x \in A)).\end{aligned}$$

Sets with other sets as members

Sets may have other sets as members. For instance, consider

$$A = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}, \quad B = \{x \mid x \text{ is a subset of } \{a, b\}\}.$$

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Note that these sets are equal: $A = B$. Note also that $\{a\}$ is a member of A, but a is not.

The size of a set

Definition

Let S be a set. If there are exactly n elements in S, where n is a non-negative integer, then we say that S is a *finite set* and that n is the *cardinality* of S, denoted by $|S|$.

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Definition

If S is not a finite set, then it is said to be *infinite*.

Power sets

Definition

Given a set S, the *power set* of S is the set of all the subsets of S. It is denoted $\mathcal{P}(S)$.

Cartesian products

Definition

The *ordered n-tuple* $(a_1, a_2, \ldots, a_n)$ is the ordered collection that has a_1 as its first element, a_2 as its second element, ..., and a_n as its nth element. When $n = 2$, these are also called *ordered pairs*.

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Equality

If $(b_1, \ldots, b_n)$ is another n-tuple, then these are said to be equal when each of their terms is equal, i.e., when $a_i = b_i$ for each choice of i. Note that the pairs (x, y) and (y, x) are equal only if $x = y$.

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Definition

A subset of the Cartesian product $A \times B$ is called a *relation* from the set A to the set B.

Using set notation with quantifiers

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Truth sets and quantifiers

Definition

Given a predicate P and domain D, we define the *truth set* of P to be the set of elements x in D for which $P(x)$ is true. This truth set is denoted

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What are the truth sets of $P(x)$, $Q(x)$, and $R(x)$, where the domain is the set of integers and

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2.2: Set Operations

Basic operations

Union

Let A and B be sets. Their union, written $A \cup B$, is the set that contains those elements which are in A, in B, or in both.

$$A \cup B = \{x \mid x \in A \vee x \in B\}.$$

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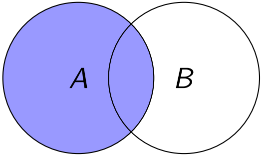
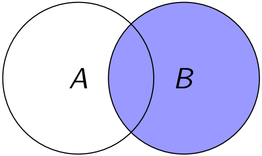


B:



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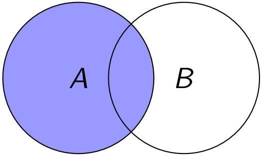

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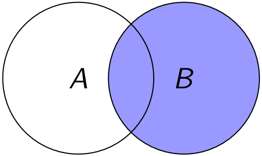

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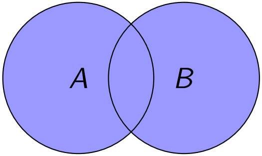

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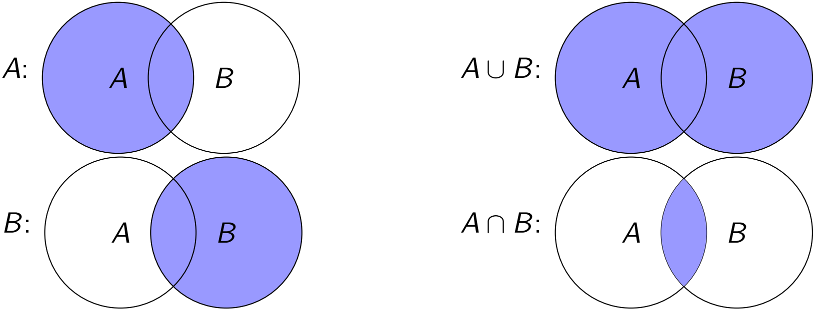

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$$|A \cup B| = |A| + |B| - |A \cap B|.$$

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Difference

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Complement

Let U be the "universal set". The complement of A, denoted $\bar{A}$, is the difference $U \setminus A$. An element x belongs to $\bar{A}$ if and only if $x \notin A$, hence:

$$\bar{A} = \{x \in U \mid x \notin A\}.$$

Basic operations

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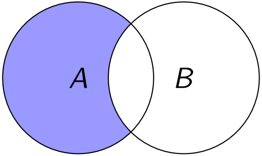

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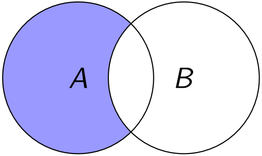

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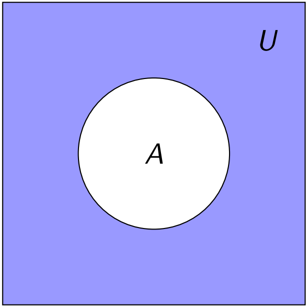

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Complementation law:

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Commutative laws:

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Associative laws:

- $A \cup (B \cup C) = (A \cup B) \cup C$,
- $A \cap (B \cap B) = (A \cap B) \cap C$.

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Distributive laws:

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$,
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap B)$.

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Set identities

Example

Use a membership table to show

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C$
0	0	0	0				
0	0	1	1				
0	1	0	1				
0	1	1	1				
1	0	0	0				
1	0	1	1				
1	1	0	1				
1	1	1	1				

Set identities

Example

Use a membership table to show

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C$
0	0	0	0	0			
0	0	1	1	0			
0	1	0	1	0			
0	1	1	1	0			
1	0	0	0	0			
1	0	1	1	1			
1	1	0	1	1			
1	1	1	1	1			

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0	0	1	1	0	0	0	
0	1	0	1	0	0	0	
0	1	1	1	0	0	0	
1	0	0	0	0	0	0	
1	0	1	1	1	0	1	
1	1	0	1	1	1	0	
1	1	1	1	1	1	1	

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0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

Generalized unions and intersections

Because unions and intersections of sets satisfy associative laws, the sets $A \cup B \cup C$ and $A \cap B \cap C$ are well defined; that is, the meaning of this notation is unambiguous. Note that $A \cup B \cup C$ contains those elements that are in at least one of the sets A, B, and C, and that $A \cap B \cap C$ contains those elements that are in all of A, B, and C.

Generalized unions and intersections

Definition

The *union of a collection of sets* is the set that contains those elements which are members of at least one set in the collection.

$$A_1 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i = \{x \mid \exists i \in \{1, \ldots, n\}(x \in A_i)\}.$$

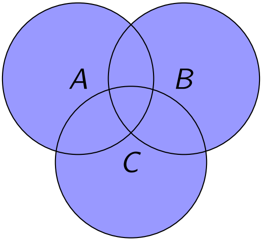
Definition

The *intersection of a collection of sets* is the set that contains those elements which are members of all the sets in the collection.

$$A_1 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i = \{x \mid \forall i \in \{1, \ldots, n\}(x \in A_i)\}.$$

Generalized unions and intersections

$A \cup B \cup C$:



Generalized unions and intersections

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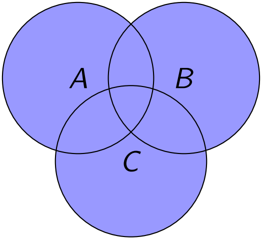

$A \cap B \cap C$: