SOLUTIONS

Math 1A Discussion — Midterm 2 Practice Problems

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1. Differentiate $y = \dfrac{x^3}{\tan(x)}$. $\boxed{y' = 3x^2\cot x - x^3\csc^2 x}$
2. Differentiate $e^{\sin(e^x)}$. $\boxed{e^{\sin(e^x)}\cos(e^x)\,e^x.}$
3. Find an equation of the line tangent to the curve

$$\ln(x+y) + 4x^3 = 4 + \ln(2) \quad \boxed{y = -25x + 26}$$

at the point $(x, y) = (1, 1)$.

4. Find $\frac{dy}{dx}$ if $1 + x^2\cos(y^2) = y^3 + e^x$. $\boxed{\dfrac{dy}{dx} = \dfrac{2x\cos(y^2) - e^x}{2x^2 y\sin(y^2) + 3y^2}}$
5. Differentiate $y = \sinh(\cosh(x))$. $\boxed{y' = \cosh(\cosh(x))\sinh(x)}$
6. Show that the equation $e^{-x} = x^3$ has exactly one solution.

Let $f(x) = e^{-x} - x^3$. $f(x)$ is continuous, $f(0) > 0$, $f(1) < 0$, so $f(x) = 0$ has a solution by the intermediate value theorem. $f'(x) = -e^{-x} - 3x^2 < 0$, so $f(x)$ has at most one solution by Rolle's theorem.

7. Find a formula for the nth derivative of $\ln(x)$. $\boxed{\dfrac{d^n}{dx^n}\ln(x) = (-1)^{n+1}(n-1)!\,x^{-n}}$
8. Find all critical numbers of $f(x) = 2x^{1/3}(3 + x^{4/3})$. $\boxed{x = 0}$
9. The half-life of silver-108 is 418 years. Find an exact expression for the number of years it takes for a 120mg sample of silver-108 to become 100mg. $\boxed{418\,\dfrac{\ln(1.2)}{\ln(2)}}$
10. Find $\displaystyle\lim_{x\to 1}\frac{\arctan(x) - 1}{x^2 - 1}$. $\boxed{\text{does not exist } \left(\lim_{x\to 1+} = -\infty,\ \lim_{x\to 1-} = \infty\right)}$
11. Verify that $f(x) = 2\sqrt{x} - x$ satisfies the three hypotheses of Rolle's theorem on the interval $[0, 4]$, and find all numbers c that satisfy the conclusion of Rolle's theorem.

- continuous on $[0,4]$ ✓
- differentiable on $(0,4)$ ✓
- $f(0) = f(4) = 0$ ✓
- $c = 1$

12. Show that $\arccos\left(\dfrac{2\sqrt{x}}{x+1}\right) = 2\arctan(\sqrt{x}) - \dfrac{\pi}{2}$ for $x \geq 1$.

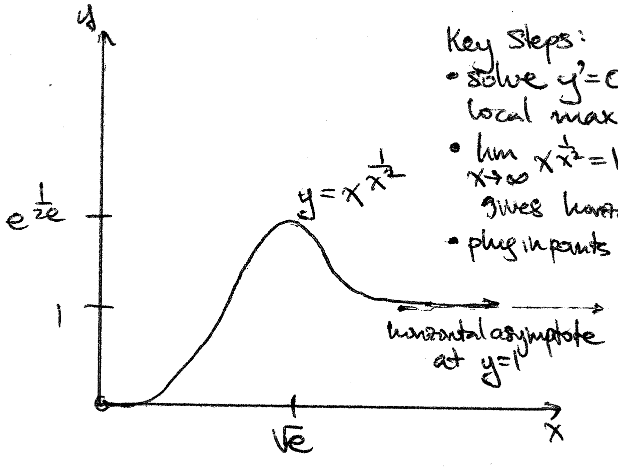
Let $f(x) = \arccos\left(\frac{2\sqrt{x}}{x+1}\right) - 2\arctan(\sqrt{x})$. $f'(x) = 0$ and $f(1) = -\frac{\pi}{2}$, so $f(x) = 0$ for all $x \geq 1$.

13. Find all critical values of $f(x) = e^x\sin(x)$. $\boxed{x = -\frac{\pi}{4} + n\pi \text{ (n an integer)}}$
14. Find all intervals on which f is increasing or decreasing and all x-values of local maxima and minima of the function $f(x) = x^2e^x$.

- increasing: $(-\infty, -2)$ and $(0, \infty)$
- decreasing: $(-2, 0)$
- maxima: $x = -2$
- minima: $x = 0$

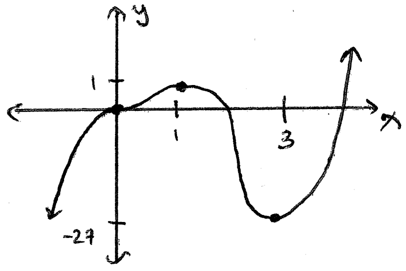
15. Sketch $y = x^{1/x^2}$ for $x > 0$.
16. Sketch the curve $y = x^5 - 5x^4 + 5x^3$.

15.



Key steps:
- solve $y' = 0$ to find local max.
- $\lim_{x\to\infty} x^{\frac{1}{x^2}} = 1$ (L'Hôpital) gives horizontal asymptote
- plug in points near $x = 0$

16.



Key steps:
- number line: 0, 1, 3; y': $+$ $+$ $-$ $+$
- plug in points