1). Shift the graph of $y = \sqrt{x}$ to the left by 3 units.
2). Let $\epsilon > 0$, and set $\delta := \sqrt{\epsilon}$. If $|x - 0| = |x| < \delta$, then $|x^2 - 0| = |x|^2 < \delta^2 = \epsilon$, which shows that $\lim_{x\to 0} x^2 = 0$.
3). Let $f(x) = e^x + x - 2$. f is continuous everywhere on $\mathbb{R}$, and $f(0) = -1$, $f(1) = e - 1 > 0$. By the Intermediate Value Theorem, f has a root in $(0, 1)$.
4). $\frac{d}{dx}\left(\frac{e^x}{x^2}\right) = \frac{x^2(e^x) - e^x(2x)}{(x^2)^2} = \frac{e^x(x-2)}{x^3}$.
5). $\frac{d}{dx}(\tan(\cos x)) = \sec^2(\cos(x))(\frac{d}{dx}(\cos x)) = -\sec^2(\cos x)\sin x$.
6. Applying $\frac{d}{dx}$ to both sides of $x^2y + xy^2 = 3x$ gives $(2xy + x^2\frac{dy}{dx}) + (y^2 + 2xy\frac{dy}{dx}) = 3$, so $\frac{dy}{dx}(x^2 + 2xy) = 3 - 2xy - y^2$, i.e. $\frac{dy}{dx} = \frac{3 - 2xy - y^2}{x^2 + 2xy}$.
7. If $f(x) = \frac{x}{x^2+1}$, $f'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(1+x^2)^2}$, so the critical numbers of f are $1, -1$. As -1 is not in in $[0, 2]$, and $f(0) = 0, f(1) = 1/2, f(2) = 2/5$, we see that $f(0) = 0$ is the absolute minimum, and $f(1) = 1/2$ is the absolute maximum, of f on $[0, 2]$.
8. Notice that $2\sin^{-1} x\big|_{x=0} = 0 = \cos^{-1}(1 - 2x^2)\big|_{x=0}$. It is thus enough to show that $\frac{d}{dx}(2\sin^{-1} x) = \frac{d}{dx}(\cos^{-1}(1 - 2x^2))$ on $[0, 1]$. Now $\frac{d}{dx}(2\sin^{-1} x) = \frac{2}{\sqrt{1-x^2}}$. On the other hand, $\frac{d}{dx}(\cos^{-1}(1 - 2x^2)) = \frac{-1}{\sqrt{1-(1-2x^2)^2}}(-4x) = \frac{4x}{\sqrt{1-(1-4x^2+4x^4)}} = \frac{2(2x)}{\sqrt{4x^2(1-x^2)}} = \frac{2(2x)}{2x\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$.
9. $\lim_{x\to 1^+} \ln(x)\tan(\pi x/2) = \lim_{x\to 1^+} \frac{\ln x}{\cot(\pi x/2)}$. As $\lim_{x\to 1^+} \ln x = 0 = \lim_{x\to 1^+} \cot(\pi x/2)$, we may apply L'Hospital's Rule, so that $\lim_{x\to 1^+} \frac{\ln x}{\cot \pi x/2} = \lim_{x\to 1^+} \frac{1/x}{-\csc^2(\pi x/2)(\pi/2)} = \frac{1}{-\pi/2} = -\frac{2}{\pi}$.

10. View the triangle in the xy-plane with one side on the x-axis, corresponding to the line segment between $(-L/2, 0)$ and $(L/2, 0)$. Then the equation describing the left side of the triangle is $y = \sqrt{3}(x + L/2)$ (and the right side is $y = -\sqrt{3}(x - L/2)$). Now, if area is to be maximized, the rectangle must be symmetric with respect to the y-axis. If the rectangle has vertices $(-a, 0)$ and $(a, 0)$, then the two other vertices must be $(-a, \sqrt{3}(-a + L/2))$ and $(a, \sqrt{3}(L/2 - a))$. Thus the area of the rectangle is $2a(\sqrt{3}(L/2 - a))$, which achieves a maximum when $a = -(L\sqrt{3})/(2(-2\sqrt{3})) = L/4$. This gives $\sqrt{3}(L/2 - a) = \sqrt{3}L/4$, so the rectangle has dimensions $L/2 \times \sqrt{3}L/4$.

11. Let (x, y) be a point on the line $y = 4x + 7$. Then the square of the distance from (x, y) to the origin is $x^2 + y^2 = x^2 + (4x + 7)^2 = 17x^2 + 56x + 49$, which has a minimum at $x = -56/(2(17)) = -28/17$. Then $(x, y) = (-28/17, 7/17)$ is the point on the line which minimizes the square of the distance, hence also minimizes the distance to the origin.

12. Let $f(x) = x^3 - 30$, so $f'(x) = 3x^2$, and $f(x)/f'(x) = x/3 - 10/x^2$. To start, try $x_0 = 3$. Newton's method gives $x_1 = 3 - f(3)/f'(3) = 3 - (3/3 - 10/3^2) = 3 - (-1/9) = 28/9$. Next, $x_2 = 28/9 - f(28/9)/f'(28/9) = 28/9 - (28/27 - 10/(28/9)^2) = 3 + 2/27 + 26/(28)^2$. The difference between these two answers is $|1/9 - 2/27 - 26/(28)^2| = |1/27 - 1/28 + 2/(28)^2| = |1/(27\cdot 28) + 2/(28\cdot 28)| < 3/(27)^2 = 1/(9\cdot 27) < 1/200 = 0.005$, so x_2 approximates $30^{1/3}$ to an accuracy of 0.005, i.e. 2 decimal places.
13. $\int (5x^{1/4} - 7x^{3/4})\,dx = 4x^{5/4} - 4x^{7/4} + C$.
14. If $f''(x) = 2 - 12x$, then $f'(x) = -6x^2 + 2x + C$, and so $f(x) = -2x^3 + x^2 + Cx + D$. Now $9 = f(0) = D$, so $15 = f(2) = -2(2)^3 + 2^2 + C(2) + 9 \Rightarrow 2C = 18 \Rightarrow C = 9$. Thus $f(x) = -2x^3 + x^2 + 9x + 9$.

15. The first rectangle will go from $x=-1$ to $x=0$, the second from $x=0$ to $x=1$, and the third from $x=1$ to $x=2$. Using right endpoints, the heights will be $f(0)=1, f(1)=2, f(2)=5$, respectively. Thus the area under $1+x^2$ between $x=-1$ and $x=2$ is approximated by the total area of the 3 rectangles, which is $1(1)+1(2)+1(5)=8$.

16. Using a right Riemann sum gives the area under $x\cos x$ from 0 to $\pi/2$ as $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\pi}{2n}(\frac{i\pi}{2n}\cos\frac{i\pi}{2n})$.

17. $\int_{-3}^{0}(1+\sqrt{9-x^2})\,dx = \int_{-3}^{0}dx + \int_{-3}^{0}\sqrt{9-x^2}\,dx = 3+\frac{\pi(3)^2}{4} = 3(1+\frac{3\pi}{4})$, as the second integral is one quarter of the area of a circle of radius 3.

18. For x in the interval $[0,\pi/4]$, $\sin x$ takes values between 0 and 1, so $\sin^3 x \le \sin^2 x$ on $[0,\pi/4]$, and thus $\int_0^{\pi/4}\sin^3 x\,dx \le \int_0^{\pi/4}\sin^2 x\,dx$.

19. By the First Fundamental Theorem of Calculus, $\frac{d}{dx}\int_0^x \sqrt{1+2t}\,dt = \sqrt{1+2x}$.

20. $\int_{\sqrt{x}}^{x^3}\sqrt{t}\sin(t)\,dt = \int_0^{x^3}\sqrt{t}\sin(t)\,dt + \int_{\sqrt{x}}^{0}\sqrt{t}\sin(t)\,dt = \int_0^{x^3}\sqrt{t}\sin(t)\,dt - \int_0^{\sqrt{x}}\sqrt{t}\sin(t)\,dt.$

By the Chain Rule, the derivative is equal to $(\sqrt{x^3}\sin(x^3))(3x^2) - (\sqrt{\sqrt{x}}\sin(\sqrt{x}))(\frac{1}{2\sqrt{x}}) = 3x^{7/2}\sin(x^3) - \frac{\sin\sqrt{x}}{2x^{1/4}}$.

21. $\int_0^2 (6x^2-4x+5)\,dx = 2x^3-2x^2+5x\big|_0^2 = 2(2)^3-2(2)^2+5(2)-0=18.$

22. $\int_0^{\pi/4}(1+\cos^2\theta)/(\cos^2\theta)\,d\theta = \int_0^{\pi/4}(\sec^2\theta+1)\,d\theta = \tan\theta+\theta\big|_0^{\pi/4} = 1+\pi/4.$

23. $\int \frac{1+4x}{\sqrt{1+x+2x^2}}\,dx \xrightarrow{u=1+x+2x^2} \int \frac{1}{\sqrt{u}}\,du = 2u^{1/2}+C = 2\sqrt{1+x+2x^2}+C.$

24. $\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx \xrightarrow{u=\sin x} \int \frac{1}{u}\,du = \ln|u|+C = \ln|\sin x|+C.$

25. $\int_0^2 (x-1)^{25}\,dx \xrightarrow{u=x-1} \int_{-1}^{1} u^{25}\,du = 0$ (u^{25} is an odd function).

26. First, notice that $\ln x = \int_1^x \frac{1}{t}\,dt$, as the derivatives of both sides equal $1/x$ (by FTC1), and both sides evaluate to 0 at $x=1$. Since $1/t$ is monotone decreasing, a right Riemann sum will give a (strict) lower bound on the area under the curve. This gives $\ln(n) =$ (area under $1/t$ from 1 to n) $>$ (right Riemann sum, with rectangles of width 1) $= 1/2+1/3+...+1/n$.

27. The curves $x=2y^2, x+y=1$ intersect when $2y^2=1-y$, i.e. $y=-1,1/2$. For $y\in[-1,1/2]$, $2y^2\le 1-y$, so the area between the curves is given by the integral (in y) $\int_{-1}^{1/2}|(1-y)-2y^2|\,dy = \int_{-1}^{1/2}(-2y^2-y+1)\,dy = -\frac{2y^3}{3}-\frac{y^2}{2}+y\Big|_{-1}^{1/2} = \frac{9}{8}$.

28. Using the disk method, a typical (horizontal) disk will have volume $\pi(\sqrt{y})^2\Delta y$, so the volume of the solid is given by $\int_0^4 \pi y\,dy = \frac{\pi y^2}{2}\big|_0^4 = 8\pi$.

29. We first find the volume of a hemisphere. A hemisphere of radius r is obtained as a solid of revolution by rotating the curve $y=\sqrt{r^2-x^2}$, $0\le x\le r$, about the y-axis. By the shell method, the volume of this solid of revolution is $\int_0^r 2\pi x\sqrt{r^2-x^2}\,dx \xrightarrow{u=r^2-x^2} \int_{r^2}^{0}-\pi\sqrt{u}\,du = \pi\int_0^{r^2}\sqrt{u}\,du = \frac{2\pi u^{3/2}}{3}\Big|_0^{r^2} = \frac{2\pi r^3}{3}$. Doubling gives the volume of a sphere of radius r, $\frac{4\pi r^3}{3}$.

30. Average value $= \frac{1}{5-2}\int_2^5 (x-3)^2\,dx = \frac{1}{3}(\frac{1}{3}(x-3)^3)\Big|_2^5 = \frac{1}{9}(2^3-(-1)^3) = 1.$