Homework #2

Math 231b

"Due": March 1st, 2017

Guidelines:

- Type up your solution to the assignment in LaTeX. (You might want to avail yourself of the excellent diagrams package `tikz-cd`.)
- Submit the PDF via Canvas, in the Assignments section.

Failure to meet these guidelines may result in loss of points.[1]

Task 1. Read Chapter 5 to see a "proper" definition of a CW-structure on a pre-existing space.

Task 2. Skim through Chapter 6 and look at all the proofs we skipped. Try reading a few. Then try reading a few more. Move on to the rest of the problem set whenever you like.

Problem 3. Show that if (X, A) is a relative CW-complex, then X/A is a CW-complex. Given CW-complexes X and Y, use this to concoct appropriate conditions so that $X \wedge Y$ is a CW-complex.

Problem 4. Suppose (X, A) is a relative CW-complex and $p\colon E \to B$ is a weak fibration. Show that for any map $f\colon X \to E$ and homotopies $F\colon X \times I \to B$, $H\colon A \times I \to E$ with $F_0 = p \circ f$, $H_0 = f|_A$, and $p \circ H = F|_{A\times I}$ there is a homotopy $G\colon X \times I \to E$ lifting F with $G|_{A\times I} = H$, $G_0 = f$, and $p \circ G = F$. Diagrammatically, these conditions are summarized as

$$\begin{array}{ccc} (X \times \{0\}) \cup (A \times I) & \xrightarrow{f \cup H} & E \\ \big\downarrow & \overset{G}{\nearrow} & \big\downarrow p \\ X \times I & \xrightarrow{F} & B. \end{array}$$

Problem 5. Suppose X is obtained from A by attending n–cells $\{e^n_\beta \mid \beta \in B\}$. Show that $X/A \cong \bigvee_{\beta\in B} S^n_\beta$, and that the homeomorphism can be chosen so that the diagram

$$\begin{array}{cccc} & (S^n, *) & \xrightarrow{i_\beta} & (\bigvee_\beta S^n_\beta, *) \\ \overset{p'}{\nearrow} & & & \\ (D^n, S^{n-1}) & & & \big\downarrow \cong \\ \underset{f_\beta}{\searrow} & & & \\ & (X, A) & \xrightarrow{p} & (X/A, *) \end{array}$$

commutes, where f_β is the characteristic map of e^n_β.

Problem 6. Show that if $f\colon X \to Y$ is a cellular map of CW-complexes, then $Y \cup_f CX$ is naturally a CW-complex.

[1] This version of the assignment was compiled on April 10, 2017.

Problem 7. Justify some of our ad hoc constructions from class by proving the following: let C be a category with finite products and a zero object and let $F \colon \mathsf{C}^{\mathrm{op}} \to \mathsf{Groups}$ be a group-valued functor. Show that if F is represented by an object Y by a natural transformation $t \colon \mathsf{C}(-, Y) \to F$, then Y carries a group structure which causes $\mathsf{C}(-, Y)$ to be group-valued and the comparison natural isomorphism t to respect the group structure.